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COUPLED DIFFUSION OF MOISTURE AND HEAT IN HYGROSCOPIC TEXTILE MATERIALS

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Abstract—A finite difference solution, based on the "double sweep" method, has been found for solving the non-linear differential equations which describe coupled diffusion of heat and mass (moisture) in hygroscopic textile materials. In addition to the diffusion equations, a rate equation has been introduced describing the rate of exchange of moisture between the solid (textile fibres) and the gas phase (pore space). A numerical application of the theory has been made using wool as an example for the hygroscopic material and it is shown that, similar to forced convective transfer, transfer of moisture from air to the wool and from the wool to air are not symmetrical processes. The magnitude of the error caused by neglecting the rate of transfer of moisture between the solid and gaseous phases as compared to the time scale of the diffusional processes is discussed and shown to depend on the physical dimensions of the slab. Examples are also given of positive and negative temperature and concentration fronts which may be set up in the slab.

NOMENCLATURE

a,	radius of a sphere [cm];
C_A,	concentration of water vapour in air filling the interfibre void space [g cm^{-3}];
C_F,	concentration of water vapour in sorbed state, expressed per unit volume of textile-air mixture [g cm^{-3}];
$C(C_F)$,	volumetric heat capacity of textile material [cal °C^{-1} cm^{-3}]. It is a function of the moisture content, C_F;
C'_A, C'_F, T',	corresponding variables for the case of a sphere;
d,	packing density of solid phase in fibre assembly [g cm^{-3}];
K,	diffusion coefficient of water vapour in air in textile assembly [cm^2 s^{-1}];
k', k_1, k_2,	rate constants for mass transfer as defined in text [s^{-1}];
k',	modified rate constant as defined in text [g cm^{-3}];
L,	thickness of slab [cm];
M,	molecular weight of water;
N,	number of integration intervals in the X-dimension;
P_A,	partial pressure of water vapour in ash [mm Hg];
$P_S(T)$,	saturation vapour pressure of water vapour [mm Hg];
Q,	thermal conductivity of the textile–air mixture [cal s^{-1} °C^{-1} cm^{-1}];
R,	universal gas constant;
r,	radial co-ordinate of a sphere [cm];
S,	parameter controlling pseudo-equilibrium;
T,	temperature of textile fibres or gas [°K];
t,	real time;
X,	dimensionless distance;
x,	real distance;
y_A, y_F,	relative humidity of air, textile fibres respectively;

Greek symbols

ε,	porosity, volume of interfibre void space divided by total volume;

$\lambda(C_F)$,	heat of sorption or desorption of water vapour by solid phase. This is a function of the concentration, C_F [$\mathrm{cal\,g^{-1}}$];
ρ,	density of the solid [$\mathrm{g\,cm^{-3}}$];
τ,	dimensionless time.

INTRODUCTION

The problem of unsteady state diffusion into an assembly of hygroscopic textile fibres was first proposed and analysed in 1939 [1] by Henry who considered the penetration of atmospheric moisture into bales of cotton. Henry showed that in this system, in which moisture diffuses into a porous structure, the solid phase of which is hygroscopic, the concentration wave entering the structure gives rise to a secondary temperature wave. This wave, in turn, diffuses (by conduction) at a rate differing, in general from that of the concentration wave.

These phenomena are of great interest in textile technology. Changes in weight of bales produced by differing atmospheric conditions are of economic significance, changes in moisture content or distribution of moisture content of cloth or yarn packages frequently affect subsequent processing and, finally, changes of temperature produced by sorption or desorption of moisture play a role in clothing physiology.

A new evaluation of the coupled diffusion problem is made here as our knowledge of the properties of textile fibres (particularly wool) has become more extensive during the intervening years. In addition, the advent of electronic computing has made it possible to solve the strongly non-linear differential equations which arise, obviating many of the simplifying assumptions which had to be made by Henry in his original work.

FORMULATION OF THE PROBLEM

Consider a small element of volume of a slab of a homogeneously packed textile material of unit area (parallel to the plane of the slab) and of thickness, δx. Water vapour is free to diffuse into the interfibre void spaces and to be sorbed or desorbed by the fibres. Volume changes of the fibres due to the changing moisture content are neglected.

The equations of conservation of mass and of heat will be expressed in terms of unit volume of the textile–air mixture. Water vapour accumulates in the unit element both in the void space and in the fibres. The total accumulation in time $\mathrm{d}t$ is given by

$$\left(\frac{\partial C_F}{\partial t} + \varepsilon\frac{\partial C_A}{\partial t}\right)\mathrm{d}t$$

where $\varepsilon = (1 - d/\rho)$. The transport of mass by diffusion is given by

$$K\frac{\partial^2 C_A}{\partial x^2},$$

the diffusion flux depending on the concentration of water vapour in the interfibre void space. Thus, the equation of conservation of mass becomes

$$K\frac{\partial^2 C_A}{\partial x^2} = \frac{\partial C_F}{\partial t} + \varepsilon\frac{\partial C_A}{\partial t}. \qquad (1)$$

It should be noted that K has not the numerical value of the diffusion coefficient of water vapour in air, but is related to it. At the present time [2] no satisfactory theory exists which relates the diffusion coefficient of a gas or vapour inside a porous body to the porosity and structure of the body. However, K may be readily determined for a given structure from steady-state diffusion measurements.

Changes in heat content of the volume element arise from several processes: conduction into or out of the element, changes of phase of water vapour (sorption or desorption), changes of temperature of the solid and of the gaseous phase. The contribution due to the last cause is very small and will be neglected. It follows that the equation of heat balance may be written

$$Q\frac{\partial^2 T}{\partial x^2} = C(C_F)\frac{\partial T}{\partial t} - \lambda(C_F)\frac{\partial C_F}{\partial t}. \qquad (2)$$

Both C, the heat capacity of the solid and λ, the energy of change of phase, are functions of the concentration of water absorbed by the solid. These relationships can be found experimentally.

Most textile fibres are of very small diameter (15–50 μ) and have a very large surface to volume ratio. Transfer of heat across the surface and conduction inside the fibres are very rapid processes. Hence, the assumption of instantaneous temperature equilibrium between solid and gaseous phases which has been made in equation (2) does not lead to an appreciable error.

In his original work, Henry [1] assumed that the transfer of water vapour from the fibres to the air surrounding them, as well as the transfer of heat, takes place instantaneously.

This assumption will lead to an appreciable error in certain circumstances and is not made in the present development. A rate equation is introduced which allows for the finite time which is required to establish moisture equilibrium.

In the development of a realistic rate equation for mass transfer a choice of several intensive variables may be made to represent the driving force determining the rate of transfer. The one chosen here is the difference between the relative humidity of the air and that of the textile fibre based on its instantaneous moisture content and temperature. The advantage of this choice arises from the equilibrium relationship (isotherm) between moisture content of the textile material and relative humidity which is essentially independent of temperature in the range of temperatures considered. The rate of moisture exchange is assumed to be proportional to the relative humidity difference. The rate equation for mass transfer may be written

$$\frac{1}{\rho(1-\varepsilon)}\frac{\partial C_F}{\partial t} = k(y_A - y_F). \qquad (3)$$

Work published over the last 10 years [3–5] has revealed that diffusion of moisture into individual wool fibres is a very complex process, the rate of diffusion for a given fibre depending, apart from minor factors, on the magnitude of the concentration change as well as the absolute value of the concentration at which the change takes place.

The process of uptake of moisture by a wool fibre appears to occur in two stages. There is a rapid first stage which exhibits the character of Fickian diffusion into a cylinder [3, 4]. This stage is followed by a slower change in moisture content which follows approximately an exponential course with respect to time. The first stage of uptake of moisture extends over a period of several minutes while the second stage may last several hours. Because of the large time constant of the second stage of uptake of moisture, it is believed to be caused by a molecular relaxation process due to the swelling of the fibre which changes the free energy of the keratin, thus causing a further increase in the moisture content.

Equation (3), therefore, is only a useful simplification of a complex phenomenon. It is used throughout this paper for devising general results. In the specific case which is discussed later, in which the present theory is tested experimentally, a more sophisticated rate equation is employed. In this equation, account is taken of the two-stage sorption behaviour of the textile material (wool) and its concentration dependence. In this case it has been found that the equation

$$\frac{1}{\rho(1-\varepsilon)}\frac{\partial C_F}{\partial t} = \left[k_2 + k_1\left(S - \frac{y_F}{y_A}\right)^2 \frac{C_F}{\rho(1-\varepsilon)}\right](y_A - y_F) \qquad (4)$$

simulates the sorption behaviour of single fibres when $y_F/y_A > S, k_1 = 0$.

k_1 and k_2 are the rate constants appropriate for the first and second stages of sorption respectively, S is a parameter controlling the pseudo-equilibrium towards which the first stage of sorption tends and

$$\frac{C_F}{\rho(1-\varepsilon)}$$

a factor to allow for the concentration dependence of the diffusion. Rate equations of this kind have to be derived from experimental data for each particular sorption system which is under consideration.

Introducing the dimensionless time

$$\tau = \frac{Kt}{L^2}$$

and the dimensionless distance

$$X = \frac{x}{L}$$

equations (1–3) become

$$\frac{\partial^2 C_A}{\partial X^2} = \frac{\partial}{\partial \tau}(C_F + \varepsilon\, C_A) \tag{5}$$

$$\frac{Q}{K}\frac{\partial^2 T}{\partial X^2} = \frac{\partial}{\partial \tau}\{C(C_F)\, T - \lambda(C_F)\, C_F\} \tag{6}$$

and

$$\frac{\partial C_F}{\partial \tau} = k'(y_A - y_F) \tag{7}$$

where

$$k' = \frac{\rho(1-\varepsilon)\, L^2 k}{K}.$$

The simultaneous partial differential equations (5–7) thus represent the formulation of the problem of coupled diffusion of heat and mass in a porous body the solid phase of which can absorb the diffusing substance according to a specified rate law.

The solution of these equations requires that y_A and y_F are expressed in terms of C_A, C_F and T. If the perfect gas law is assumed

$$y_A = \frac{P_A}{P_S(T)}\, C_A\, T \frac{R}{M} \tag{8}$$

where $P_S(T)$ is the saturation vapour pressure of water (in the present case) and is a function of temperature only. A numerical expression for this relationship is given in the Appendix.

The equilibrium relative humidity of the fibres, y_F, is related to the concentration of water sorbed by the textile material and can be expressed by the isotherm

$$y_F = f\left[\frac{C_F}{\rho(1-\varepsilon)}\right]. \tag{9}$$

Relationships of this kind are known for most textile materials, a numerical representation of the isotherm for wool is given in the Appendix.

The porosity, ε, for a given textile assembly can be determined accurately and for most practical cases lies in the range 0·7–0·95.

The ratio of the thermal conductivity of the fibre–air mixture to the diffusion coefficient of water vapour in the mixture, i.e. Q/K, can be obtained from independent measurement. Usually Q is about 20–50 per cent [7] higher than the thermal conductivity of air. K, the diffusion coefficient inside the porous structure is less than the corresponding value in air and may vary typically from 90 to 50 per cent of the value in air [8]. Thus, the variation of magnitude of the ratio Q/K encountered in practical examples does not exceed 3 to 1.

The rate constant, k' is chosen so as to represent as closely as possible the single fibre sorption kinetics.

The functional relationships for the heat capacity of the moist textile material, $C(C_F)$ and the energy of sorption, $\lambda(C_F)$ depend on the type of textile used. For wool, these relationships are given in Appendix.

METHOD OF SOLUTION

Initially, we are interested in the simultaneous solution of equations (5–7) for diffusion into a slab of finite thickness. Because of the non-linear character of the equations, the method of finite differences was adopted for their solution. Following closely the scheme mentioned in Richtmeyer [9] as due to Laasonen [10], equation (5), for example, becomes

$$\begin{aligned}&\frac{\delta\tau}{(\delta X)^2}[C_A(X + \delta X, \tau + \delta\tau)\\&\quad - 2\,C_A(X, \tau' + \delta\tau) + C_A(X - \delta X, \tau + \delta\tau)]\\&\quad = [C_F(X, \tau + \delta\tau) - C_F(X, \tau)]\\&\qquad + \varepsilon\,[C_A(X, \tau + \delta\tau) - C_A(X, \tau)]. \qquad (10)\end{aligned}$$

Equations (6) and (7) can be formulated similarly. The functions $C(C_F)$ and $\lambda(C_F)$ in equation (6) are evaluated at the mesh point (X, τ) while a more refined approximation is used to evaluate the functions y_A and y_F in equation (7) at the mesh point $(X, \tau + \delta\tau)$. This is done by taking the derivative of these functions at the point (X, τ) and extrapolating to $(X, \tau + \delta\tau)$. For the function y_A, this becomes

$$\begin{aligned}&y_A(X, \tau + \delta\tau) = y_A(X, \tau) + \left(\frac{\partial y_A}{\partial C_A}\right)_X,\\&\quad \times \tau[C_A(X, \tau + \delta\tau) - C_A(X, \tau)]\\&\quad + \left(\frac{\partial y_A}{\partial T}\right)_X \tau[T(X, \tau + \delta\tau) - T(X, \tau)] \qquad (11)\end{aligned}$$

and a similar expression (without the time derivative) is obtained for y_F. The derivatives are found from equations (8) and (9).

In solving equations (5–7) in the finite difference form it is convenient first to substitute equation (7) into (5) and (6) so that two equations in C_A and T are obtained. Once these are evaluated, C_F follows directly from (7).

If the slab of textile material of unit dimensionless thickness is subdivided into N sections of equal thickness, δX, there will be $N + 1$ points on the mesh in the X-dimension and hence $N + 1$ values of any one of the variables at any one time, τ.

Since equations (5) and (6) in the finite difference form [such as (10)] contain three of these consecutive variables each, a total of $2N - 2$ equations must be set up across the slab. As the boundary conditions on each side of the slab give each two values of the variables, namely

$$C_A(0, \tau), T(0, \tau) \quad \text{and} \quad C_A(X, \tau), T(X, \tau),$$

the total number of variables is reduced to $2N - 2$ for the $2N - 2$ equations. A solution can thus be obtained from a conventional matrix inversion.

It has been shown by Richtmeyer (11), however, that a more economical method of solving the set of simultaneous equations exists if a linear recurrence relationship can be found between terms denoting adjoining space intervals. This is the case in the present problem and Richtmeyer's simplified scheme is applicable.

If the initial condition of the slab is given by

$$C_A(N\delta X, 0); \quad C_F(N\delta X, 0) \quad \text{and} \quad T(N\delta X, 0) \quad \text{for all } N\delta X \text{ at } \tau = 0$$

and a new boundary condition imposed, then

$$C_A(N\delta X, 1); C_F(N\delta X, 1) \quad \text{and} \quad T(N\delta X, 1)$$

can be calculated as just described for $\tau = 1$ and all values of N from 1 to $N - 1$. This process can be iterated for as many time intervals as required. The solution of the equations was found to be stable for all values of δX and $\delta\tau$.

For a mesh of 10000 points and $N = 20$ the computing time using a CDC 3600 computer was about 80 s.

The solution of the coupled diffusion problem for a parallel slab can be adapted easily to the corresponding case of diffusion into a sphere. For diffusion into a sphere, equations (5) and (6) become

$$\frac{\partial^2 C_A}{\partial r^2} + \frac{2\partial^2 C_A}{r\,\partial r^2} = \frac{\partial}{\partial\tau}\{C_F^{+}\ \varepsilon \cdot C_A\} \qquad (12a)$$

and

$$\frac{Q}{K}\left\{\frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r}\right\} = \frac{\partial}{\partial\tau}\{C(C_F)\,T - \lambda(C_F)\,C_F\}. \qquad (12b)$$

Equation (7) remains unchanged except that

$$k' = \frac{\rho(1 - \varepsilon)\,a^2 k}{K}.$$

It can be shown that by making the substitution

$$C_A = \frac{C'_F}{r}; \quad C_F = \frac{C'_A}{r} \quad \text{and} \quad T = \frac{T'}{r}$$

equations (12a) and (12b) become identical in form to the original equations (5) and (6) and hence the method of solution is identical to that for the slab. Since at the centre of the sphere, $r = 0$, the boundary conditions for the new variables become

$$C'_A(0, \tau) = 0; \quad C'_F(0, \tau) = 0 \quad \text{and} \quad T'(0, \tau) = 0.$$

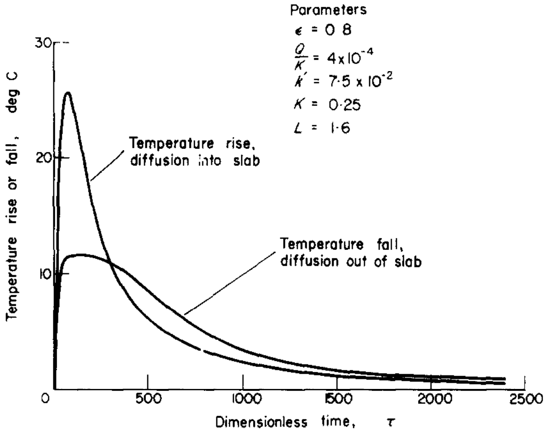

FIG. 1. Temperature in centre of slab. Boundary conditions change from 0 to 80 per cent and from 80 to 0 per cent relative humidity.

GENERAL DEDUCTIONS FROM THE NUMERICAL SOLUTIONS

Since the diffusion problem discussed in this paper deals with a highly non-linear system, a numerical evaluation of the theory must, of necessity, take recourse to a specific example. Thus, use is made of the perfect gas law and the dependence of the saturation vapour pressure of water on temperature for the diffusate and of the equilibrium isotherm, energy of sorption and specific sorption kinetics for the stationary phase which, in the present example is wool (Appendix).

In discussing the results, particular attention is paid to Henry's earlier work [1] because the present theory may be regarded as an alternate treatment. Brief mention is also made of an experimental application of the theory using a slab of wool felt.

ASYMMETRY OF DIFFUSION INTO, OR OUT OF, SPECIMEN

In Fig. 1, the temperature changes in the central plane of a slab of wool are shown for diffusion into, or out of, the slab.

For diffusion into the slab, the wool was taken to be initially dry, at room temperature, and suddenly subjected to a change in which the relative humidity of the air on both sides of the slab was raised to 80 per cent. For diffusion out of the slab, the wool was taken to be initially in equilibrium with air at 80 per cent relative humidity and at room temperature. It was then subjected to a change in which the relative humidity on both sides of the slab was reduced to 0 per cent. A pronounced asymmetry is noted which is similar to, but not quite as marked as that observed in the case of forced convective [12] heat and mass transfer and which is believed to be due mainly to the non-linearity of the saturation vapour pressure curve of water [13]. This behaviour is not predicted by Henry's original theory as he used a linearized relationship between concentration of sorbed water, concentration of diffusing water vapour and temperature. The present model also predicts different rates of weight change for the two cases, as shown in Fig. 2, although this result presupposes that the kinetics of sorption and desorption of individual fibres are similar.

INFLUENCE OF SIZE OF SLAB AND SORPTION KINETICS

The parameter, k', in equation (7) which controls the influence of the rate of sorption on the coupled diffusion of heat and mass depends not only on the rate constant for sorption by individual fibres but also on the size of the specimen, L, and the diffusion coefficient, K. Given k for a particular fibre type, a large value of L (or small value of k) produces a large value of k'. Since $\partial C_F/\delta\tau$ must be finite, an infinitely large value of k' requires that $y_A = y_F$ and corresponds to the assumption of instantaneous equilibrium between air and fibres with respect to mass transfer. In Fig. 3 the influence of the parameter, k', is shown on the temperature in the centre of the slab, all other parameters being equal. When k' is sufficiently large, its influence decreases and the overall process is diffusion controlled. For low values of k' (small specimen) the process is limited by the sorption kinetics of the solid phase.

INFLUENCE OF THERMAL AND MASS CONDUCTIVITIES

The value of the parameter, Q/K,* has a pronounced influence when the overall process is diffusion controlled and a small one when controlled by the rate of sorption. This is shown in Fig. 4 where the temperature in the centre of the slab is plotted for two values of Q/K at three different rates.

Unfortunately no data are available for the variation of Q/K for different textile assemblies. However, it should not deviate greatly from a value of $2{\cdot}3 \times 10^{-4}\,\mathrm{cal\,cm^{-3}\,{}^{\circ}C^{-1}}$ at room temperature.

EFFECT OF NON-LINEARITY OF ISOTHERM AND ENERGY OF SORPTION

A numerical evaluation of diffusion into the slab for sequential and cumulative humidity steps is shown in Figs. 5 and 6 respectively, the temperature at the centre of the slab taken as the dependent variable. The shapes of the curves for the sequential steps do not vary markedly suggesting that the influence of the curvature of the isotherm is not very large.

The more pronounced change in shape of the curves in Fig. 6 is probably due to a cumulative effect depending on the size of the step. It is not believed to be due to the form of the rate equation (7) as, for the example given, the process was chosen to be mainly diffusion controlled. Similar considerations apply to diffusion out of the slab, although the calculated curves are not given here.

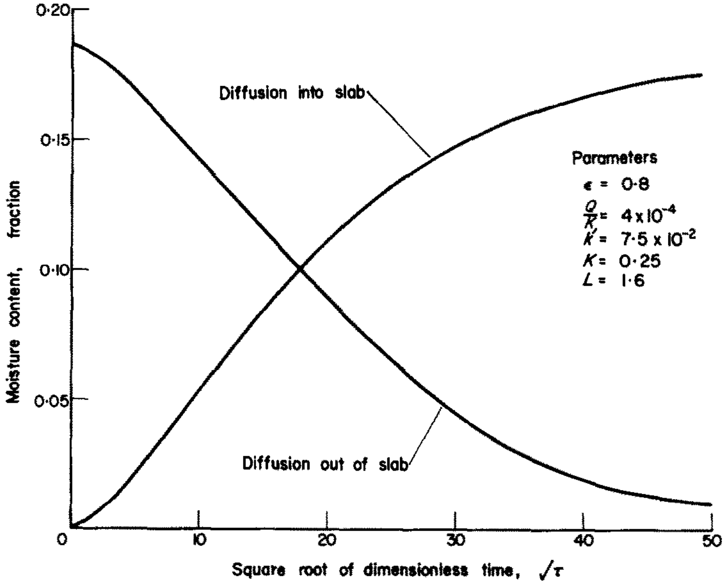

FIG. 2. Total moisture content of slab. Boundary conditions change from 0 to 80 per cent and from 80 to 0 per cent relative humidity.

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* It may have been more satisfactory to define Q as the thermal diffusivity, making Q/K dimensionless. This was thought inadvisable as the heat capacity of the solid phase in this problem is variable. As defined, Q/K is a constant for a given textile assembly and does not depend on moisture content.

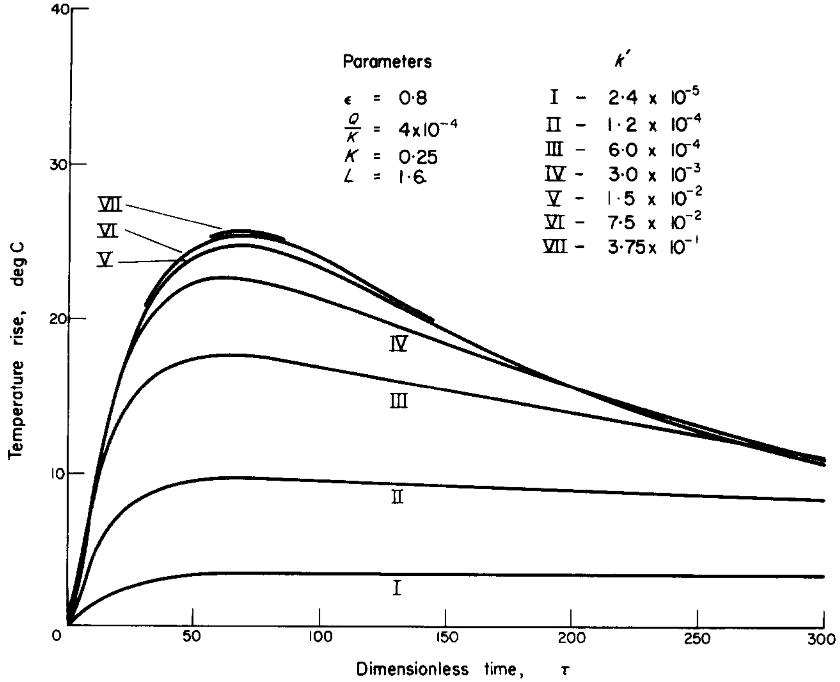


FIG. 3. Temperature in centre of slab. Change from 0 to 80 per cent relative humidity for various thicknesses of slab.

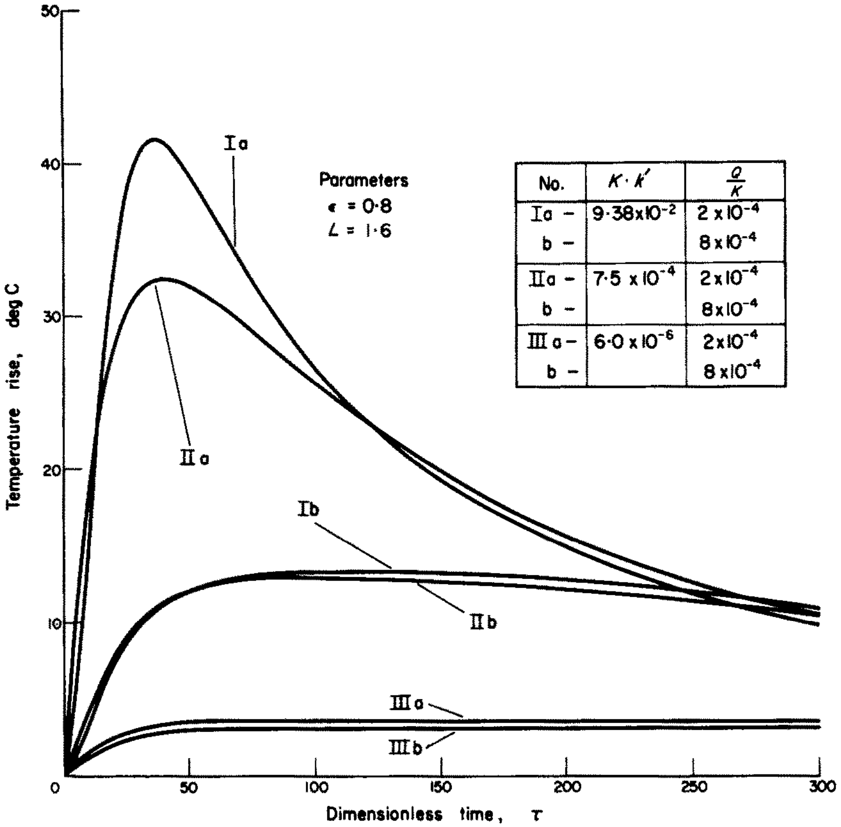
SPATIAL DISTRIBUTION OF TEMPERATURE AND MOISTURE CONTENT, TOTAL QUANTITY OF MOISTURE GAINED OR LOST

There are no special features which the present system exhibits with regard to the spatial distribution of temperature and moisture content in the slab which may not be inferred from ordinary diffusion. In the case where the boundary conditions are changed from 0 to 80 per cent and 80 to 0 per cent relative humidity, results for spatial distributions at various values of τ are given in Fig. 7. For the same case, the distribution of moisture in the slab is plotted as a function of distance in Fig. 8.

FORMATION OF DIFFUSION FRONTS

Perhaps the most interesting aspect of the coupled diffusion problem is the formation of fronts of temperature, concentration of water vapour, the absorbed moisture content, etc., which result from specified changes in the boundary conditions. Henry [1] predicted from his theory of coupled diffusion that a change in the boundary conditions would, in general, give rise to two sets of diffusion fronts or waves. One set consists of a vapour concentration and an associated temperature front which moves relatively fast and is associated mainly with heat transfer and the other set, also consists of a concentration and temperature front which is relatively slow and associated with mass transfer. The relative amplitudes of these fronts may be positive or negative and will vary in magnitude, depending on the change in boundary conditions.

The present finite difference evaluation of the basic differential equations fully agree with Henry's conclusions. For example, in the case in which the relative humidity is changed from



No.	$K \cdot k'$	$\frac{Q}{K}$
Ia –	$9{\cdot}38 \times 10^{-2}$	2×10^{-4}
b –		8×10^{-4}
IIa –	$7{\cdot}5 \times 10^{-4}$	2×10^{-4}
b –		8×10^{-4}
IIIa –	$6{\cdot}0 \times 10^{-6}$	2×10^{-4}
b –		8×10^{-4}

FIG. 4. Temperature at centre of slab. Change from 0 to 80 per cent relative humidity. Varying thermal conductivities for three thicknesses of slab.

0 to 80 per cent without change in external temperature, the fast temperature front is represented by a rise in temperature, reaching completion at about $\tau = 72$ (Fig. 1) and the slow front by a drop in temperature of equal amplitude, so that finally, when both fronts have passed, the original temperature is regained. Of the two corresponding vapour concentration curves, only the slow one is observed as the fast one is very small and superimposed on the major one.

A subsidiary fast moisture concentration front is shown in Fig. 9 for the same slab of wool where the boundary conditions are changed by keeping the absolute humidity external to the slab constant and changing the temperature. The two temperature fronts are both positive and shown in Fig. 10.

Another change of boundary condition, which is not evaluated here, is a change of external temperature, keeping the total moisture content of the fibre assembly constant (Walker [14]).

EXPERIMENTAL VERIFICATION

A detailed experimental verification of the theory has not been attempted to date. However, measurements of the temperature in the central plane of a slab of felt of density $0{\cdot}264$ g cm^{-3} and $\frac{1}{2}$-in thick were made when air at 0 and 80 per cent relative humidity was blown over it at 20°C. Difficulties in keeping the boundary layer thin in relation to the slab and, at the same time not

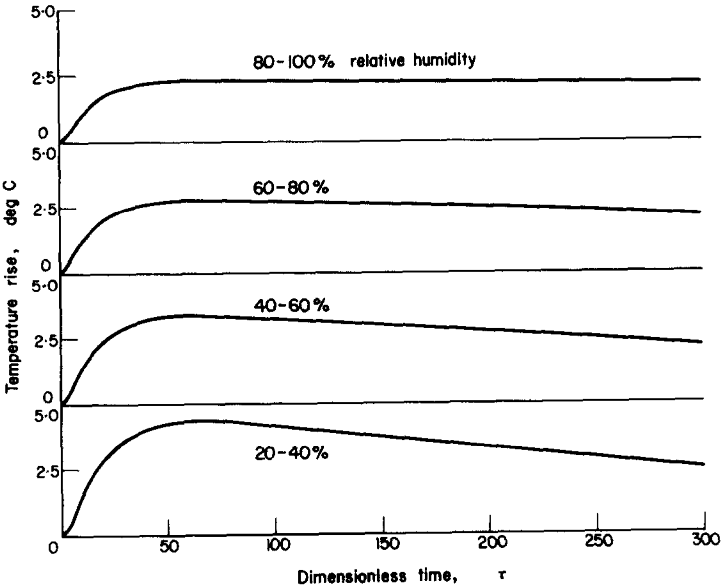


FIG. 5. Temperature at centre of slab for four sequential steps in relative humidity.

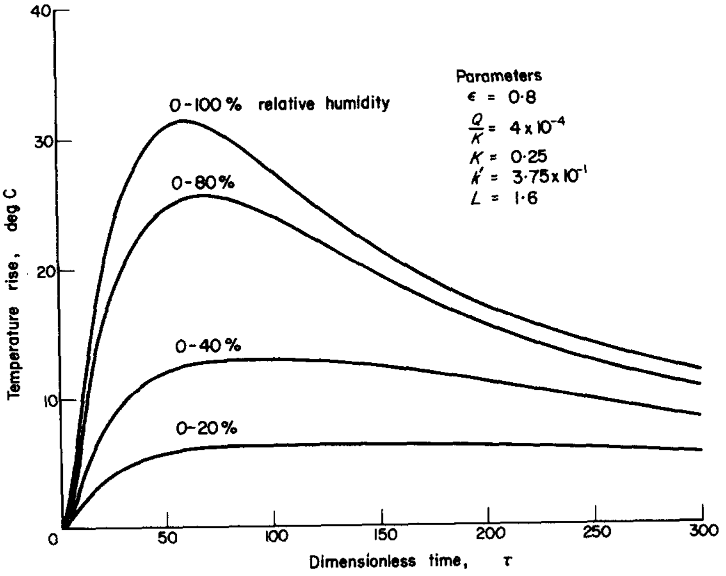


FIG. 6. Temperature at centre of slab for four cumulative steps of relative humidity.

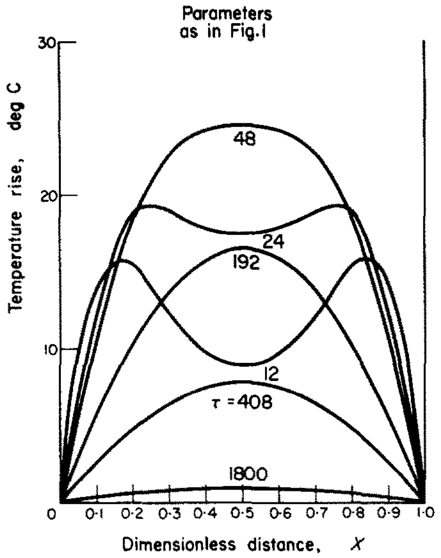


FIG. 7. Spatial distribution of temperature inside slab. Change from 0 to 80 per cent relative humidity.

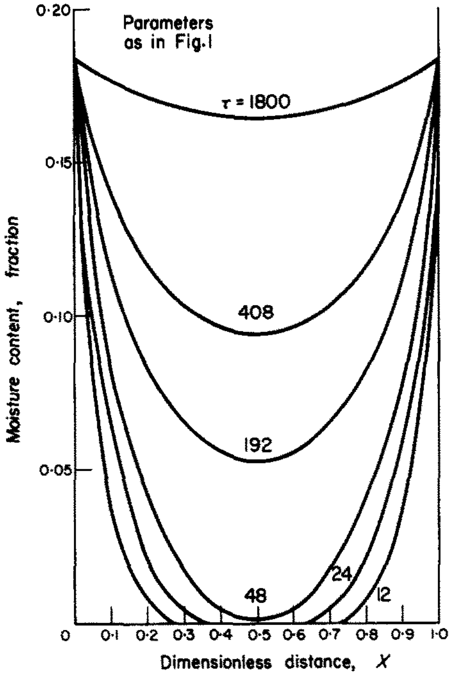


FIG. 8. Spatial distribution of moisture content inside slab. Change from 0 to 80 per cent relative humidity.

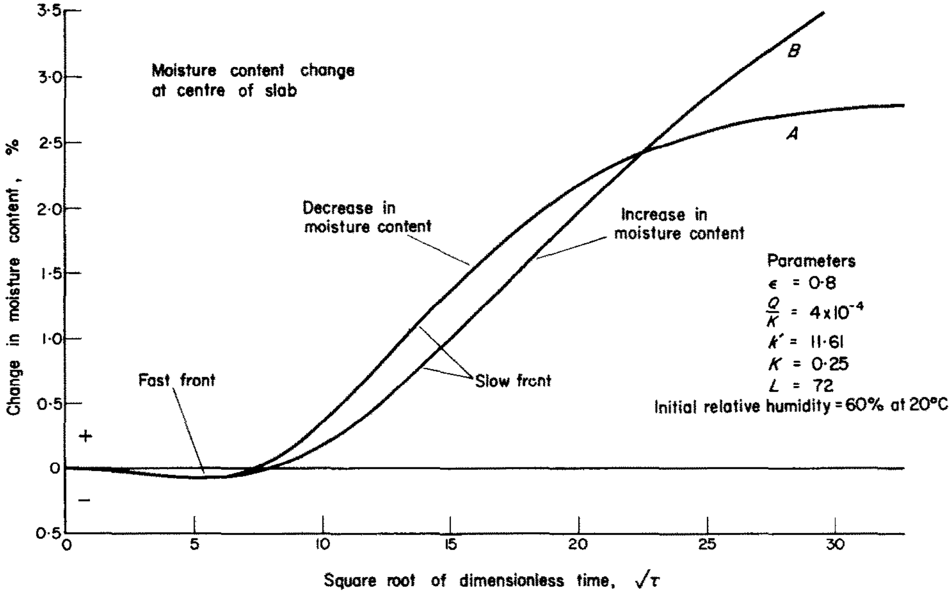


FIG. 9. Change of moisture content at centre of slab.

A. Slab initially at 20°C, 60 per cent R.H., then temperature of surrounding increased by 5 degC.
B. Same initial conditions, temperature decreased 5 degC.

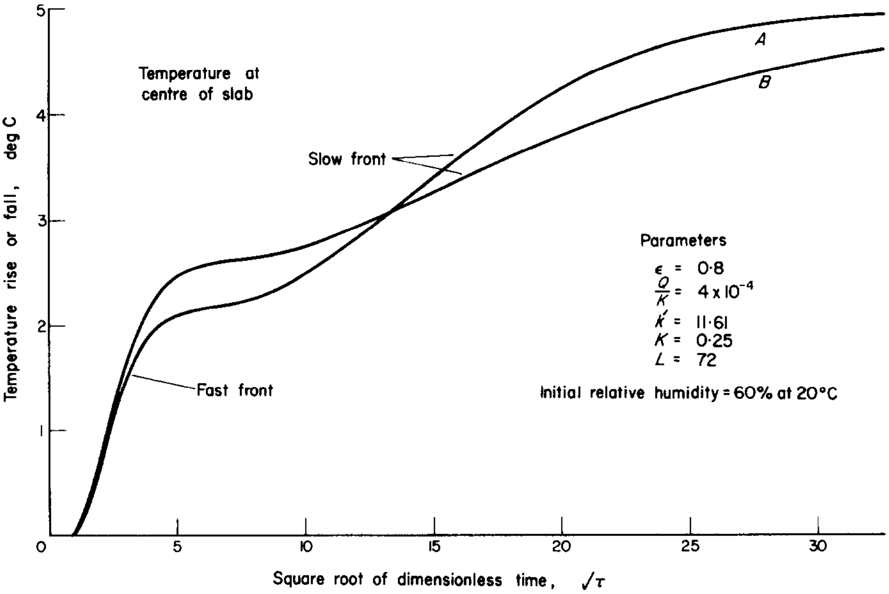


FIG. 10. Temperature at centre of slab for the same conditions as given in Fig. 9.

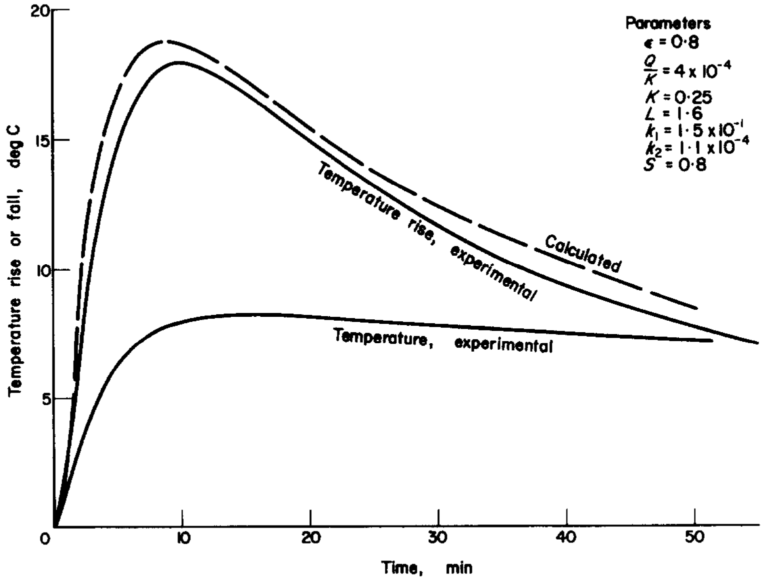


FIG. 11. Temperature at centre of slab. Change from 0 to 80 per cent and from 80 to 0 per cent relative humidity. Comparison between experiment and theory.

allowing any convective transfer to occur, make the results somewhat uncertain. It is seen from Fig. 11 that the general shape of the curves support the predicted asymmetry (Fig. 1) between diffusion into, or out of, the felt. A direct comparison between Figs. 11 and 1 is not appropriate. While a simple rate equation [equation (7)] was used in calculating Fig. 1, the more complicated equation (4) had to be used to obtain better agreement with experiment as indicated by the dotted line in the figure.

The tentative conclusions to be drawn from this experiment and the application of the theory to the present case are: 1. Diffusion processes into, and out of, the slab are not symmetrical,* 2. In cases where the overall process is partially rate (of sorption) controlled, a fairly detailed knowledge of the sorption kinetics is desirable and a simplified rate equation such as (7) may not suffice. For many purposes, however, equation (7) may be adequate. Changes of moisture content, for example are not as critically affected as temperature.

CONCLUSION

The coupled diffusion of heat and mass which is treated here by a finite difference solution of the basic differential equations presents certain advantages and certain disadvantages in comparison with Henry's analytical method. Among the advantages are that an allowance can be made to account for the finite rate of sorption of the solid phase (textile fibres) and that the magnitude of the effect can be judged. Also, a better method of incorporating the non-linear properties of the system is possible and hence the calculation of changes in boundary conditions over a wider range should be feasible. On the other hand, the method does not give the physical insight into the problem which the analytical solution affords. Hence one may conclude that the two methods are complementary.

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APPENDIX

Numerical Data for Wool Used in Present Calculations

$$P_S(T) = 4{\cdot}5855 + 0{\cdot}32808\,(T - T_0) + 0{\cdot}1172 \times 10^{-1}\,(T - T_0)^2 + 0{\cdot}12793 \times 10^{-3}\,(T - T_0)^3 + 0{\cdot}41848 \times 10^{-5}\,(T - T_0)^5$$

where $T_0 = 273{\cdot}2\,^{\circ}\mathrm{K}$

$$y_F = 1{\cdot}0232 \exp\left\{-4{\cdot}81291 \exp \times \left[-16{\cdot}191\left(\frac{C_F}{\rho(1-\varepsilon)}\right)\right]\right\} - 0{\cdot}0083$$

$$\lambda(C_F) = 1809\left(\frac{C_F}{\rho(1-\varepsilon)} - 0{\cdot}3292\right)^2 + 593$$

$$C(C_F) = 0{\cdot}32\,\rho(1-\varepsilon) + C_F \qquad [15].$$

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* This is not because of sorption hysteresis or differences in the energies of sorption or desorption, but due to nonlinearities in the system.

Résumé —On a découvert une solution par différences finies, basée sur la méthode de "double balayage", pour résoudre les équations différentielles non linéaires qui décrivent la diffusion couplée de la chaleur et de masse (humidité) dans les matériaux textiles hygroscopiques. En plus des équations de diffusion, on a introduit une équation de vitesse décrivant la vitesse d'échange d'humidité entre le solide (fibres textiles) et la phase gazeuse (espace des pores). On a fait une application numérique de la théorie en employant la laine comme exemple pour le matériau hygroscopique et l'on montre que de la même façon que le transport par convection forcée, les transports de l'humidité de l'air à la laine et de la laine à l'air ne sont pas des processus symétriques. La grandeur de l'erreur provoquée par la négligence du transport d'humidité entre les phases solide et gazeuse en comparaison avec l'échelle de temps des processus de diffusion est discutée et l'on montre qu'elle dépend des dimensions physiques de la plaque. On donne également des exemples de fronts de température et de concentration positifs et négatifs qui peuvent s'établir dans la plaque.

Zusammenfassung—Mit Hilfe endlicher Differenzen wurde auf Grund der "Double Sweep Methode" eine Lösung für die nichtlineare Differentialgleichung gefunden, die den gekoppelten Wärme- und Stoff- (Feuchtigkeits-) übergang in hygroskopischen Textilmaterialien beschreiben. Zusätzlich zu den Übergangsgleichungen wurde eine weitere Gleichung eingeführt, die den Austausch von Feuchtigkeit zwischen der festen (Textilfasern) und der gasförmigen Phase (Porenraum) angibt. Eine numerische Anwendung der Theorie wurde an Wolle als hygroskopisches Material durchgeführt, wobei gezeigt wird, dass ähnlich wie bei Zwangskonvektion die Übergänge der Feuchtigkeit von Luft an Wolle und von Wolle an Luft nicht symmetrische Prozesse darstellen. Die Grösse des Fehlers, der durch Vernachlässigung des Feuchtigkeitstransportes zwischen festen und gasförmigen Phasen im Vergleich mit dem Zeitmasstab der Übergangsprozesse entsteht wird diskutiert, und es wird seine Abhängigkeit von den Dimensionen der Probe gezeigt. Es werden auch Beispiele gegeben für positive und negative Temperatur- und Konzentrationsfronten, die sich in der Probe ausbilden.

Аннотация—Методом «двойного выметания» получено решение в конечных разностях нелинейных дифференциальных уравнений для описания совместной диффузии тепла и массы (влаги) в гигроскопических текстильных материалах. Кроме уравнений диффузии использовалось уравнение скорости влагообмена между твердым теплом (волокна ткани) и газовой фазой. (пористое пространство). Применение теории проиллюстрировано численным примером, где в качестве гигроскопического материала взята шерсть, и показано, что перенос влаги от воздуха к шерсти и от шерсти к воздуху, аналогичный массообмену при вынужденной конвекции, не является симметричным процессом. Величина погрешности в результате пренебрежения скоростью влагообмена между твердой и газовой фазами рассматривается в сравнении со шкалой времени диффузионных процессов, и показано, что эта величина зависит от физических размеров плиты. Приводятся примеры положительных и отрицательных фронтов температуры и концентрации, возможных в плите.